3 VARIANTS OF POSSIBLE EXAMINATIONS

APPLIED MATHEMATICS 10/11

Variant 1

Problem 1: Find the domain of the function $f(x) = \sqrt{\dfrac{x^2+x-12}{x^2-25}} + \ln(64-x^2)$.

Problem 2: Compute and simplify the first derivative of the function

$$f(x) = \ln\sqrt{\frac{3-\sin 3x}{3+\sin 3x}}.$$

Problem 3: Compute $\displaystyle\int \frac{\ln x}{2\sqrt{x}}\,\mathrm{d}x$.

Problem 4: Evaluate area of the plane region bounded by the curves

$$y = x^2 - 2, \quad y = 2x + 1.$$

Problem 5: Compute the matrix $\boldsymbol{X}$ from the matrix equation $\boldsymbol{AX} - \boldsymbol{B} = \boldsymbol{A}$, if

$$\boldsymbol{A} = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}.$$

Problem 6: Find the value of x, if

$$\begin{vmatrix} -1 & 1 & 3 \\ 3 & x & -9 \\ 2 & 5 & x-6 \end{vmatrix} = \begin{vmatrix} 2 & x \\ x & -3 \end{vmatrix}.$$

Variant 2

Problem 1: Find the domain of the function $f(x) = \dfrac{1}{x-2} + \arccos\frac{2x-1}{5} + \log(x^2-1)$.

Problem 2: Determine equations of the tangent and normal lines to the graph of the function

$$f : y = \frac{\sqrt{3x^2+4x+2}}{x} \quad \text{in the point } T = [1, ?] \text{ of its graph.}$$

Problem 3: Find maximal intervals on which the function f is convex or concave,

$$f(x) = x + \operatorname{arctg}(2x+3).$$

Problem 4: Compute

$$\int \left(\frac{\operatorname{arctg} x}{1+x^2} + \frac{2}{\sqrt[3]{x}} \right) \mathrm{d}x.$$

Problem 5: Determine all solutions (x, y, z, t) of the following system

$$\begin{array}{rcrcrcrcr} 3x & - & y & + & 2z & + & 3t & = & 7, \\ x & + & y & + & z & & & = & 6, \\ -2x & + & y & - & 2z & - & 3t & = & -6, \\ -x & - & 2y & + & z & + & 3t & = & -2. \end{array}$$

Problem 6: Evaluate the derminant

$$\begin{vmatrix} 3 & 1 & 0 & -1 \\ 2 & 1 & 1 & 2 \\ -1 & 2 & 1 & -2 \\ 1 & 3 & 2 & 1 \end{vmatrix}.$$

Variant 3

Problem 1: Find the domain of the function $f(x) = \ln(x^2 - 4) + \sqrt{\dfrac{x^2 - 9}{x^2 - x - 20}}$.

Problem 2: Find maximal intervals on which the function f is decreasing or increasing

$$f(x) = x - 2\ln x + \sqrt{2}.$$

Problem 3: Evaluate area of the plane region bounded by the curves

$$x = -1, \quad y = e^{-2x}, \quad y = e^{x}.$$

Problem 4: Find the general solution of the differential equation

$$xy' = 3y.$$

Problem 5: Compute the matrix $\boldsymbol{X}$ from the matrix equation $\boldsymbol{XB} - \boldsymbol{A} = 2\boldsymbol{X} + 2\boldsymbol{A}$, if

$$\boldsymbol{A} = \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} 4 & 1 \\ -3 & 0 \end{pmatrix}.$$

Problem 6: By the Cramer rule find the value y from the system

$$\begin{array}{rcrcrcr} 2x & - & y & + & 3z & = & -9, \\ 4x & + & 2y & + & 3z & = & 0, \\ -x & + & 4y & + & 6z & = & 0. \end{array}$$

Results.

1/1) $(-8, -5) \cup \langle -4, 3\rangle \cup (5, 8)$

1/2) $\dfrac{9\cos 3x}{\sin^2 3x - 9}$

1/3) $\sqrt{x}(\ln x - 2) + C$

1/4) $\frac{32}{3}$

1/5) $\boldsymbol{X} = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$

1/6) $x = 2$

2/1) $\langle -2, -1) \cup (1, 2) \cup (2, 3\rangle$

2/2) $t: 4x + 3y = 13$, $n: 3x - 4y = -9$

2/3) convex on $(-\infty, -\frac{3}{2}\rangle$, concave on $\langle -\frac{3}{2}, \infty)$

2/4) $\frac{1}{2}\operatorname{arctg}^2 x + \frac{3}{2}x\sqrt[3]{x} + C$

2/5) $(1, 2 + t, 3 - t, t)$

2/6) -6

3/1) $(-\infty, -4) \cup \langle -3, -2) \cup (2, 3\rangle \cup (5, \infty)$

3/2) increasing on $\langle 2, \infty)$, decreasing on $(0, 2\rangle$

3/3) $\frac{e^2 - 3}{2} + \frac{1}{e}$

3/4) $y = Cx^3$

3/5) $\boldsymbol{X} = \begin{pmatrix} 15 & 9 \\ 12 & 6 \end{pmatrix}$

3/6) $y = 3$