5 Indefinite integral

(Applied Mathematics — FAPPZ)

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1 Antiderivative

1.1 Definition of antiderivative

Definition 1. Let f and F be functions defined on the interval $\mathcal{J}$. Function F is *antiderivative* to the function f on $\mathcal{J}$, if, for each $x \in \mathcal{J}$, $F'(x) = f(x)$. If the interval $\mathcal{J}$ contains some of its endpoints, then at this endpoint we consider the corresponding one sided derivative.

1.2 Existence of antiderivative

Theorem 2. *Let f be continuous on interval $\mathcal{J}$, then it has an antiderivative F on $\mathcal{J}$.*

This theorem has a simple corollary.

Corollary 3. *If f is an elementary function defined on interval $\mathcal{J}$, then it has an antiderivative F on $\mathcal{J}$.*

1.3 Indefinite integral

Theorem 4. *Let F be an antiderivative to f on interval $\mathcal{J}$. Then a function G, $G(x) = F(x) + C$, $x \in \mathcal{J}$, where $C \in \mathbb{R}$ is a constant, is also an antiderivative to f on interval $\mathcal{J}$.*

Theorem 5. *Let functions F and G be antiderivatives to f on interval $\mathcal{J}$. Then there exists a constant $C \in \mathbb{R}$ such that, for any $x \in \mathcal{J}$, $G(x) = F(x) + C$.*

Definition 6 (Indefinite integral)**.** The set of all antiderivatives to f on interval $\mathcal{J}$ is called *indefinite integral* of function f on interval $\mathcal{J}$. We use the notation

$$\int f(x)\ \mathrm{d}x = F(x) + C,$$

where the symbol $\mathrm{d}x$ indicates that x is a variable in the function f, $F(x)$ is one of *antiderivatives to* $f(x)$ on $\mathcal{J}$ and $C \in \mathbb{R}$ is *an arbitrary constant.*

2 Computing indefinite integrals

2.1 Rules for integration

Theorem 7. *Let functions f, g have antiderivatives on $\mathcal{J}$. Then functions $(f+g)$, $(f-g)$ and $c\,f$, $c \in \mathbb{R}$, have also antiderivatives on $\mathcal{J}$ and*

$$\int (f(x) + g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx,$$
$$\int (f(x) - g(x))\,dx = \int f(x)\,dx - \int g(x)\,dx,$$
$$\int cf(x)\,dx = c\int f(x)\,dx.$$

Important!
There are no rules for products, quotients or composite functions.

2.2 Method of direct integration

We use only basic formulae, rules for integration and manipulation with expressions.

2.3 Method of integration by parts

Theorem 8 (Integration by parts)**.** *Let functions u and v have continuous derivatives u' and v', respectively, on interval $\mathcal{J}$. Then*

$$\int u(x)v'(x)\,dx\ = u(x)v(x) - \int u'(x)v(x)\,dx.$$

2.4 Change of variables

Theorem 9 (Change of variables)**.** *Let function f be continuous on interval $\mathcal{I}$ and F be its antiderivative on $\mathcal{I}$. Moreover, assume that function g has the first derivative $g'(x)$ at each point $x \in \mathcal{J}$ and $g(x) \in \mathcal{I}$ for any $x \in \mathcal{J}$. Then the composite function $F(g(x))$ is an antiderivative to $f(g(x))g'(x)$ on interval $\mathcal{J}$.*