11) MATRIX OPERATIONS, THE INVERSE TO A MATRIX, MATRIX EQUATIONS

APPLIED MATHEMATICS (FAPPZ)

Basic. For two matrices $\boldsymbol{A}$, $\boldsymbol{B}$ determine whether $\boldsymbol{A} = \boldsymbol{B}$:

1)
$$\boldsymbol{A} = \begin{pmatrix} \frac{3}{6} & 0 & -2 \\ 4 & \frac{1}{\sqrt{2}} & -1 \\ 1 & 1 & -3 \end{pmatrix}, \qquad \boldsymbol{B} = \begin{pmatrix} \frac{1}{2} & \ln 1 & -2 \\ 4 & \frac{\sqrt{2}}{2} & -\operatorname{tg}\frac{\pi}{4} \\ \ln e & 1 & -3 \end{pmatrix},$$

2)
$$\boldsymbol{A} = \begin{pmatrix} 1 & 2 & -2 \\ 3 & 2 & -1 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \boldsymbol{B} = \begin{pmatrix} 1 & 2 & -2 \\ 3 & 2 & -1 \end{pmatrix}.$$

3) Compute $3\boldsymbol{A} - 2\boldsymbol{B} + \boldsymbol{C}$ for
$$\boldsymbol{A} = \begin{pmatrix} 2 & -1 & 1 \\ 0 & -1 & 4 \end{pmatrix}, \qquad \boldsymbol{B} = \begin{pmatrix} -2 & 1 & 3 \\ -5 & 2 & 2 \end{pmatrix}, \qquad \boldsymbol{C} = \begin{pmatrix} -4 & 3 & 5 \\ -5 & 7 & 1 \end{pmatrix}.$$

4) Find a matrix $\boldsymbol{X}$ as a solution of the matrix equation $3\boldsymbol{X} - 2\boldsymbol{B} = 5\boldsymbol{A} + \boldsymbol{C}$, where
$$\boldsymbol{A} = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}, \qquad \boldsymbol{B} = \begin{pmatrix} -2 & 1 \\ 5 & 4 \end{pmatrix}, \qquad \boldsymbol{C} = \begin{pmatrix} 5 & 2 \\ 4 & 1 \end{pmatrix}.$$

Compute a product of matrices:

5) $$\begin{pmatrix} 7 & 4 \\ 5 & 1 \end{pmatrix} \begin{pmatrix} 5 & -2 \\ -3 & -1 \end{pmatrix},$$

6) $$\begin{pmatrix} 2 & 1 & -1 \\ 1 & 0 & 1 \\ -2 & -3 & 1 \\ 5 & 2 & 3 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -3 & -2 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

From examinations. 7) To a given matrix compute the inverse by the Jordan elimination and verify the result
$$\begin{pmatrix} 5 & -2 & 0 \\ 3 & -3 & 2 \\ 1 & -3 & 3 \end{pmatrix}.$$

8) Find a matrix $\boldsymbol{X}$ from the matrix equation $\boldsymbol{AXB} = \boldsymbol{C}$ with
$$\boldsymbol{A} = \begin{pmatrix} 3 & -2 \\ -5 & 3 \end{pmatrix}, \quad \boldsymbol{B} = \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}, \quad \boldsymbol{C} = \begin{pmatrix} -7 & 1 \\ 5 & 2 \end{pmatrix}.$$

9) Find a matrix $\boldsymbol{X}$ from the matrix equation $\boldsymbol{AX} + 3\boldsymbol{B} = 2\boldsymbol{X} + 5\boldsymbol{B}$ with
$$\boldsymbol{A} = \begin{pmatrix} 1 & 7 \\ 1 & -6 \end{pmatrix}, \qquad \boldsymbol{B} = \begin{pmatrix} -1 & -3 \\ 2 & 1 \end{pmatrix}.$$

Results.

1) $\boldsymbol{A} = \boldsymbol{B}$

2) $\boldsymbol{A} \in \mathbf{M}_{33}, \boldsymbol{B} \in \mathbf{M}_{23} \Rightarrow \boldsymbol{A} \neq \boldsymbol{B}$

3) $\begin{pmatrix} 6 & -2 & 2 \\ 5 & 0 & 9 \end{pmatrix}$

4) $\boldsymbol{X} = \begin{pmatrix} \frac{11}{3} & \frac{19}{3} \\ \frac{19}{3} & 3 \end{pmatrix}$

5) $\begin{pmatrix} 23 & -18 \\ 22 & -11 \end{pmatrix}$

6) $\begin{pmatrix} 5 \\ 2 \\ 11 \\ 0 \end{pmatrix}$

7) $\begin{pmatrix} 3 & -6 & 4 \\ 7 & -15 & 10 \\ 6 & -13 & 9 \end{pmatrix}$

8) $\boldsymbol{X} = \begin{pmatrix} 9 & -8 \\ \frac{31}{2} & -\frac{53}{4} \end{pmatrix}$

9) $\boldsymbol{X} = \begin{pmatrix} -12 & 34 \\ -2 & 4 \end{pmatrix}$