12) DETERMINATS, CRAMER'S RULE

APPLIED MATHEMATICS (FAPPZ)

Basic. Evaluate the following determinants.

$$1)\ \begin{vmatrix} 3 & 2 \\ -9 & -5 \end{vmatrix} \quad 2)\ \begin{vmatrix} 1 & 4 \\ 2 & 7 \end{vmatrix} \quad 3)\ \begin{vmatrix} 3 & 1 & 2 \\ 2 & 0 & 7 \\ 1 & -4 & -5 \end{vmatrix} \quad 4)\ \begin{vmatrix} -2 & 3 & 24 \\ 10 & 2 & -46 \\ 2 & -5 & -4 \end{vmatrix}$$

$$5)\ \begin{vmatrix} 5 & 2 & 1 & 3 \\ 2 & 3 & 3 & 2 \\ 7 & 1 & 1 & 2 \\ 5 & 3 & 0 & 5 \end{vmatrix} \quad 6)\ \begin{vmatrix} 8 & 1 & 3 & -2 \\ 2 & 4 & 1 & 2 \\ 5 & 1 & 2 & -3 \\ 2 & 5 & 7 & -5 \end{vmatrix}$$

From examinations. Solve an equation and an inequality.

$$7)\ \begin{vmatrix} 1 & 1 & 1 \\ 1 & (1-x) & 1 \\ 1 & 1 & (2-x) \end{vmatrix} > 0 \quad 8)\ \begin{vmatrix} 1 & 0 & 5 \\ 2 & (x+2) & 7 \\ 1 & 3x & x \end{vmatrix} \leq \begin{vmatrix} -1 & 2x \\ x & 13 \end{vmatrix}$$

Using determinants compute an inverse to a given matrix.

$$9)\ \boldsymbol{A} = \begin{pmatrix} -1 & 1 & 1 \\ 0 & -2 & -1 \\ -3 & 1 & 1 \end{pmatrix} \quad 10)\ \boldsymbol{A} = \begin{pmatrix} 1 & -3 & 1 \\ 3 & -3 & -1 \\ 3 & -5 & 1 \end{pmatrix}$$

Solve the following system of linear equations by the Cramer rule.

$$11)\ \begin{aligned} 2x + 5y + z &= -2 \\ x - 3y - 4z &= 1 \\ 3x + 4y + 2z &= 5 \end{aligned} \qquad 12)\ \begin{aligned} 2x - 3y + z &= 2 \\ x + 2y - 5z &= 6 \\ x - 4y + 10z &= 3 \end{aligned}$$

Results.

1) 3 2) -1 3) 85

4) -976 5) -6 6) 333

7) $x \in (-\infty, 0) \cup (1, \infty)$ 8) $x = -1$ 9) $\boldsymbol{A}^{-1} = \frac{1}{2}\begin{pmatrix} 1 & 0 & -1 \\ -3 & -2 & 1 \\ 6 & 2 & -2 \end{pmatrix}$

10) $\boldsymbol{A}^{-1} = \frac{1}{2}\begin{pmatrix} -4 & -1 & 3 \\ -3 & -1 & 2 \\ -3 & -2 & 3 \end{pmatrix}$ 11) $(x, y, z) = (3, -2, 2)$ 12) $(x, y, z) = (5, 3, 1)$