3) DIFFERENTIATION OF COMPOSITE FUNCTIONS, EQUATIONS OF TANGENT AND NORMAL

APPLIED MATHEMATICS (FAPPZ)

1. Differentiation of composite functions

Compute the first derivative $f'(x)$ of the function $y = f(x)$.

Basic.

1) $y = \sqrt{1-x^2}$ 2) $y = \sqrt[3]{a+bx^3}$ 3) $y = \arcsin 2x$

4) $y = (3x^2+2x)^5$ 5) $y = \operatorname{arctg} \frac{1}{x}$ 6) $y = 5e^{-x^2}$

From examinations.

7) $y = \operatorname{arctg}^2 \frac{1}{x}$ 8) $y = \ln \sqrt{\frac{x-1}{x+1}}$ 9) $y = \arcsin \frac{1-x}{1+x}$

10) $y = x \arccos x - \sqrt{1-x^2}$ 11) $y = 4\sqrt[4]{x} - 4 \operatorname{arctg} \sqrt[4]{x}$ 12) $y = \dfrac{1+\cos 2x}{1-\cos 2x}$

Advanced.

13) $y = -\dfrac{\arcsin x}{x} + \ln \frac{1-\sqrt{1-x^2}}{x}$ 14) $y = \ln \frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}-x}$ 15) $y = 2 \operatorname{arctg} x - \dfrac{\ln(x^2+1)}{x}$

Compute the second derivative $f''(x)$ of the function $y = f(x)$.

16) $y = \dfrac{2x+3}{5x-1}$ 17) $y = \dfrac{1-\ln x}{1+\ln x}$ 18) $y = \dfrac{3+e^{2x}}{5-e^{2x}}$

Results.

1) $\dfrac{-x}{\sqrt{1-x^2}}$ 2) $\dfrac{bx^2}{\sqrt[3]{(a+bx^3)^2}}$ 3) $\dfrac{2}{\sqrt{1-4x^2}}$

4) $5(3x^2+2x)^4(6x+2)$ 5) $\dfrac{-1}{1+x^2}$ 6) $-10xe^{-x^2}$

7) $-\dfrac{2 \operatorname{arctg} \frac{1}{x}}{1+x^2}$ 8) $\dfrac{1}{x^2-1}$ 9) $\dfrac{-1}{\sqrt{x}(1+x)}$

10) $\arccos x$ 11) $\dfrac{1}{\sqrt[4]{x}(1+\sqrt{x})}$ 12) $-2\dfrac{\cos x}{\sin^3 x}$

13) $\dfrac{\arcsin x}{x^2}$ 14) $\dfrac{2}{\sqrt{x^2+a^2}}$ 15) $\dfrac{\ln(x^2+1)}{x^2}$

16) $\dfrac{170}{(5x-1)^3}$ 17) $\dfrac{2(3+\ln x)}{x^2(1+\ln x)^3}$ 18) $\dfrac{32e^{2x}(5+e^{2x})}{(5-e^{2x})^3}$

2. Equations of tangent and normal

Basic. 1) Find an equation of the tangent line to the graph of the function $f : y = x^2 - 3x + 1$ at the point of its graph $T = [2, y_T]$.

2) Find an equation of the normal line to the graph of the function $f : y = \dfrac{x+1}{x+3}$ at the point of its graph $T = [-2, y_T]$.

From examinations. 3) Find equations of the tangent and the normal to the graph of the function $f : y = \dfrac{x}{1-\cos x}$ at the point of its graph $T = [\pi, y_T]$.

4) Find equations of the tangent and the normal to the graph of the function $f : y = \operatorname{arccotg} \frac{1+x}{1-x}$ at the point of its graph $T = [0, y_T]$.

Advanced. 5) Find an equation of the tangent to the curve $y = x^2 - 1$ so that the tangent is parallel with line $p : 2x - y + 3 = 0$.

6) Find an equation of the normal to the curve $y = x^2$ so that the normal is parallel with line $p : 2x - 6y + 5 = 0$.

7) Find an equation of the tangent to the curve $y = x^2$ so that the tangent is perpendicular to line $p : 2x - 6y + 12 = 0$.

8) In which point is the curve $y = \dfrac{1}{x^2 - 4x + 5}$ parallel with x-axis?

9) Find an equation of the normal to the curve $y = \ln \frac{e^{x+1}}{x^2+1}$ so that the normal is parallel with line $p : 2x + 4y + 5 = 0$.

Results.

1) $t : y = x - 3$
2) $n : y = -\frac{1}{2}x - 2$
3) $t : y = \frac{1}{2}x, \quad n : y = -2x + \frac{5}{2}\pi$
4) $t : y = -x + \frac{\pi}{4}, \quad n : y = x + \frac{\pi}{4}$
5) $t : y = 2x - 2$
6) $n : y = \frac{1}{3}x + \frac{5}{6}$
7) $t : y = -3x - \frac{9}{4}$
8) $A = [2, 1]$
9) $n : x + 2y + 1 + \ln 4 = 0$