4) MONOTONY, EXTREME VALUES

APPLIED MATHEMATICS (FAPPZ)

Basic. Given a function $f : y = f(x)$. Find maximal intervals on which this function is monotone.

$$1)\quad y = 2 + x - x^2 \qquad 2)\quad y = x^5 - 15x^3 + 3$$

From examinations. Find maximal intervals on which $f : y = f(x)$ is monotone.

$$3)\quad y = 2x^2 - \ln x \qquad 4)\quad y = \arcsin\sqrt{4 - x^2}$$

$$5)\quad y = \ln\tfrac{2x+3}{3x-1} \qquad 6)\quad y = \operatorname{arctg} x - \ln x$$

For $f : y = f(x)$ find local extreme values.

$$7)\quad y = 2 + 5\,e^{\sqrt{16-x^2}} \qquad 8)\quad y = x - \frac{2x+1}{3x-5}$$

$$9)\quad y = \frac{1+\ln x}{x} \qquad 10)\quad y = \operatorname{arctg} x - \ln\sqrt{1+x^2}$$

Advanced. For $f : y = f(x)$ find, on a given interval, all its extreme values (i. e. local and global). Conclude whether they are sharp. If the interval is not given, do it on the domain of f.

$$11)\quad y = -2\cdot 10^{2-6x-x^2} + \log 2,\ x \in \langle -4, 0\rangle \qquad 12)\quad y = \sqrt{2x - x^2}$$

$$13)\quad y = \arcsin\sqrt{2 - x} \qquad 14)\quad y = \operatorname{arctg}\sqrt{1 - x^2}$$

Results. 1) increasing on $(-\infty, \frac{1}{2}\rangle$, decreasing on $\langle \frac{1}{2}, \infty)$

2) increasing on $(-\infty, -3\rangle$, $\langle 3, \infty)$ decreasing on $\langle -3, 3\rangle$

3) increasing on $\langle \frac{1}{2}, \infty)$, decreasing on $(0, \frac{1}{2}\rangle$

4) increasing on $\langle -2, -\sqrt{3}\rangle$, decreasing on $\langle \sqrt{3}, 2\rangle$

5) decreasing on $(-\infty, -\frac{3}{2})$, $(\frac{1}{3}, \infty)$

6) decreasing on $(0, \infty)$

7) increasing on $\langle -4, 0\rangle$, decreasing on $\langle 0, 4\rangle$

8) no local extreme values

9) sharp local minimum $f(1) = 1$, no local maximum

10) sharp local maximum $f(1) = \frac{\pi}{4} - \ln\sqrt{2}$, no local minimum

11) sharp global maximum $f(0) = \log 2 - 200$, sharp global and local minimum $f(-3) = \log 2 - 2\cdot 10^{11}$, no local maximum

12) sharp global and local maximum $f(1) = 1$, global minimal values (not sharp) $f(0) = f(2) = 0$, no local minimum

13) sharp global maximum $f(1) = \frac{\pi}{2}$, sharp global minimum $f(2) = 0$, no local extreme values

14) sharp global and local maximum $f(0) = \frac{\pi}{4}$, global minimal values (not sharp) $f(-1) = f(1) = 0$, no local minimum