

5) CONVEXITY, CONCAVITY, INFLECTION POINTS

APPLIED MATHEMATICS (FAPPZ)

Basic. Determine maximal intervals of convexity and concavity of $f : y = f(x)$.

- 1) $y = 5x^2 + 20x + 7$ 2) $y = x(1 - x)^2$
 3) $y = 3x^5 - 5x^4 + 3x - 2$ 4) $y = x^4 - 10x^3 + 36x^2 + x - \ln 2$

From examinations. Determine maximal intervals of convexity and concavity and inflection points of $f : y = f(x)$.

- 5) $y = \frac{1}{x^2 - 1}$ 6) $y = \frac{x}{\ln x}$
 7) $y = \operatorname{arctg} \frac{1}{x}$ 8) $y = \ln(x^2 - 1)$
 9) $y = \frac{x^3}{x^2 + 12}$ 10) $y = e^{\operatorname{arctg} x}$

Advanced. Determine maximal intervals of monotony and of convexity and concavity and inflection points of $f : y = f(x)$.

- 11) $y = \sqrt[3]{x^3 - 6x^2}$ 12) $y = \arcsin \frac{2x}{x^2 + 1}$

Results. 1) convex on $(-\infty, \infty)$

- 2) concave on $(-\infty, \frac{2}{3})$, convex on $(\frac{2}{3}, \infty)$
 3) concave on $(-\infty, 1)$, convex on $(1, \infty)$
 4) concave on $(-\infty, 1)$, convex on $(1, \infty)$
 5) concave on $(-1, 1)$, convex on $(-\infty, -1)$, $(1, \infty)$, no points of inflection
 6) concave on $(0, 1)$, (e^2, ∞) , convex on $(1, e^2)$, inflection point $[e^2, e^2/2]$
 7) concave on $(-\infty, 0)$, convex on $(0, \infty)$, no points of inflection
 8) concave on $(-\infty, -1)$, $(1, \infty)$, no points of inflection
 9) concave on $(-6, 0)$, $(6, \infty)$, convex on $(-\infty, -6)$, $(0, 6)$, inflection points $[-6, -9/2]$, $[0, 0]$, $[6, 9/2]$
 10) concave on $(\frac{1}{2}, \infty)$, convex on $(-\infty, \frac{1}{2})$, inflection point $[\frac{1}{2}, e^{\operatorname{arctg} \frac{1}{2}}]$
 11) increasing on $(-\infty, 0)$, $(4, \infty)$, decreasing on $(0, 4)$, concave on $(6, \infty)$, convex on $(-\infty, 0)$, $(0, 6)$
 12) increasing on $(-1, 1)$, decreasing on $(-\infty, -1)$, $(1, \infty)$, concave on $(-\infty, -1)$, $(-1, 0)$, convex on $(0, 1)$, $(1, \infty)$