

6) INDEFINITE INTEGRAL

APPLIED MATHEMATICS (FAPPZ)

For being skilful in computing indefinite integrals it is necessary to compute sufficiently many exercises.

Basic (practising various methods). Compute indefinite integral $\int f(x) dx$.

Method of direct integration.

$$\begin{array}{ll} 1) \int \left(2x^3 - \frac{x}{2} + \frac{1}{x}\right) dx & 2) \int \left(\frac{1}{\sqrt[4]{x^3}} + \frac{1}{\sqrt{x}}\right) dx \\ 3) \int \frac{(1 + \sqrt{x})^2}{6x} dx & 4) \int 10^x e^x dx \\ 5) \int \frac{(2^x - 3^x)^2}{6^x} dx & 6) \int \cotg^2 x dx \\ 7) \int \frac{5 \operatorname{tg}^2 x - 1}{\sin^2 x} dx & 8) \int \left(\sin \frac{x}{2} - \cos \frac{x}{2}\right)^2 dx \end{array}$$

Change of variables. In the following exercises use substitution for an expression at the argument of a trigonometric function.

$$9) \int \frac{dx}{\sin^2 3x} \quad 10) \int \cos \frac{2x-3}{5} dx \quad 11) \int x^3 \sin x^4 dx$$

In the following exercises substitute for the denominator of a function.

$$12) \int \frac{dx}{1-x} \quad 13) \int \frac{e^x dx}{2+e^x} \quad 14) \int \frac{x^3 dx}{x^2-1}$$

In the following exercises substitute for an expression at brackets.

$$15) \int \frac{x dx}{\sqrt{(x^2-1)^3}} \quad 16) \int \frac{x^2 dx}{(1-x^3)^2} \quad 17) \int \frac{x^3 dx}{(2+3x^2)^3}$$

In the following exercises substitute for an expression inside the root (or for the whole root).

$$18) \int \frac{x dx}{\sqrt{1-x}} \quad 19) \int \frac{e^{2x} dx}{\sqrt{e^x-1}} \quad 20) \int \frac{dx}{\cos^2 x \sqrt{\operatorname{tg} x - 1}}$$

In the following exercises substitute for an expression at the exponent.

$$21) \int e^{-x} dx \quad 22) \int x^4 e^{4x^5+7} dx \quad 23) \int \frac{10^{\cotg x}}{\sin^2 x} dx$$

In the following exercises substitute for a logarithmic function.

$$24) \int \frac{dx}{x \ln^7 x} \quad 25) \int \frac{\ln(x+1) dx}{x+1} \quad 26) \int \frac{dx}{x(1+\ln^2 x)}$$

In the following exercises substitute for a suitable trigonometric function.

$$27) \int \frac{\operatorname{tg}^7 x dx}{\cos^2 x} \quad 28) \int \frac{\cos x dx}{\sin^5 x} \quad 29) \int \frac{\sin x dx}{\cos^3 x}$$

In the following exercises use change of variables to obtain $\int \frac{dt}{t^2+1} = \operatorname{arctg} t + C$.

$$30) \int \frac{dx}{16+25x^2} \quad 31) \int \frac{e^x dx}{1+e^{2x}} \quad 32) \int \frac{x dx}{4+x^4}$$

In the following exercises use change of variables to obtain $\int \frac{dt}{\sqrt{1-t^2}} = \arcsin t + C$.

$$33) \int \frac{dx}{\sqrt{16-9x^2}} \quad 34) \int \frac{dx}{\sqrt{3-5x^2}} \quad 35) \int \frac{x dx}{\sqrt{1-x^4}}$$

Integration by parts. Direct application of integration by parts.

$$36) \int x^2 \sin x dx \quad 37) \int \frac{\ln x dx}{\sqrt{x}} \quad 38) \int \frac{x^2 dx}{e^x}$$

Integrals which after integration by parts lead to an equation.

$$39) \int e^x \sin x dx \quad 40) \int \sin^2 x dx \quad 41) \int \frac{\ln x dx}{x}$$

From examinations. Integrals of various types.

$$\begin{aligned} 42) \int \frac{\sqrt{x}}{1+\sqrt{x}} dx & \quad 43) \int \sin x \sqrt{2+\cos x} dx & \quad 44) \int \arccos x dx \\ 45) \int \left(x \sqrt{x^2-9} + \frac{1}{\sqrt{x}} \right) dx & \quad 46) \int \left((3-2x)e^x + \frac{2}{x} \right) dx & \quad 47) \int \frac{dx}{2\sqrt{x}(x+1)} \\ 48) \int 3x \left(\sin x^2 + \frac{2}{x^2} \right) dx & \quad 49) \int \left(\frac{\operatorname{arctg} x}{1+x^2} + \frac{2}{\sqrt[3]{x}} \right) dx & \quad 50) \int \frac{1+\sin \sqrt{x}}{\sqrt{x}} dx \end{aligned}$$

Advanced. Integrals of various types.

$$51) \int \frac{\cos^3 x dx}{1-\sin x} \quad 52) \int x \operatorname{arctg}^2 x dx$$

Results.

$$\begin{aligned} 1) \frac{x^4}{2} - \frac{x^2}{4} + \ln|x| + C & \quad 2) 4\sqrt[4]{x} + 2\sqrt{x} + C \\ 3) \frac{1}{6} \ln|x| + \frac{2}{3}\sqrt{x} + \frac{1}{6}x + C & \quad 4) \frac{(10e)^x}{1+\ln 10} \\ 5) \frac{\left(\frac{2}{3}\right)^x - \left(\frac{3}{2}\right)^x}{\ln 2 - \ln 3} - 2x + C & \quad 6) -\operatorname{cotg} x - x + C \\ 7) 5 \operatorname{tg} x + \operatorname{cotg} x + C & \quad 8) x + \cos x + C \\ 9) -\frac{\operatorname{cotg} 3x}{3} + C & \quad 10) \frac{5}{2} \sin \frac{2x-3}{5} + C \\ 11) -\frac{\cos x^4}{4} + C & \quad 12) -\ln|1-x| + C \\ 13) \ln(2+e^x) + C & \quad 14) \frac{x^2-1}{2} + \frac{1}{2} \ln|x^2-1| + C \\ 15) \frac{-1}{\sqrt{x^2-1}} + C & \quad 16) \frac{1}{3(1-x^3)} + C \\ 17) \frac{-(3x^2+1)}{18(2+3x^2)^2} + C & \quad 18) -\frac{2}{3}(2+x)\sqrt{1-x} + C \\ 19) \frac{2}{3}(e^x+2)\sqrt{e^x-1} + C & \quad 20) 2\sqrt{\operatorname{tg} x - 1} + C \\ 21) -e^{-x} + C & \quad 22) \frac{e^{4x^5+7}}{20} + C \end{aligned}$$

- 23) $\frac{-10^{\cotg x}}{\ln 10} + C$ 24) $\frac{-1}{6 \ln^6 x} + C$
- 25) $\frac{\ln^2(x+1)}{2} + C$ 26) $\operatorname{arctg}(\ln x) + C$
- 27) $\frac{\operatorname{tg}^8 x}{8} + C$ 28) $\frac{-1}{4 \sin^4 x} + C$
- 29) $\frac{1}{2 \cos^2 x} + C$ 30) $\frac{1}{20} \operatorname{arctg} \frac{5x}{4} + C$
- 31) $\operatorname{arctg} e^x + C$ 32) $\frac{1}{4} \operatorname{arctg} \frac{x^2}{2} + C$
- 33) $\frac{1}{3} \arcsin \frac{3x}{4} + C$ 34) $\frac{\sqrt{5}}{5} \arcsin \left(x \sqrt{\frac{5}{3}} \right) + C$
- 35) $\frac{1}{2} \arcsin x^2 + C$ 36) $2x \sin x + (2 - x^2) \cos x + C$
- 37) $2\sqrt{x}(\ln x - 2) + C$ 38) $-(x^2 + 2x + 2)e^{-x} + C$
- 39) $\frac{1}{2}e^x(\sin x - \cos x) + C$ 40) $\frac{1}{4}(2x - \sin 2x) + C$
- 41) $\frac{1}{2} \ln^2 x + C$ 42) $x - 2\sqrt{x} + 2 \ln(1 + \sqrt{x}) + C$
- 43) $-\frac{2}{3} \sqrt{(2 + \cos x)^3} + C$ 44) $x \arccos x - \sqrt{1 - x^2} + C$
- 45) $\frac{1}{3}(x^2 - 9)^{3/2} + 2\sqrt{x} + C$ 46) $(5 - 2x)e^x + 2 \ln|x| + C$
- 47) $\operatorname{arctg} \sqrt{x} + C$ 48) $-\frac{3}{2} \cos x^2 + 6 \ln x + C$
- 49) $\frac{1}{2} \operatorname{arctg}^2 x + \frac{3}{2} x^{4/3} + C$ 50) $2\sqrt{x} - 2 \cos \sqrt{x} + C$
- 51) $\sin x + \frac{\sin^2 x}{2} + C$ 52) $\frac{(x^2 + 1) \operatorname{arctg}^2 x}{2} - x \operatorname{arctg} x + \ln \sqrt{x^2 + 1} + C$