

8) DEFINITE INTEGRAL AND ITS APPLICATIONS

APPLIED MATHEMATICS (FAPPZ)

Basic (various methods).

Direct integration.

$$1) \int_1^2 \frac{\sqrt{x\sqrt{x}}}{\sqrt[3]{x}} dx \quad 2) \int_{\pi/6}^{\pi/3} \frac{dx}{\sin^2 x \cos^2 x} \quad 3) \int_0^1 \frac{e^{2x} + e^x}{e^x} dx$$

Integration by parts. Direct application of integration by parts.

$$4) \int_0^\pi x \sin x dx \quad 5) \int_1^3 \ln x dx \quad 6) \int_0^1 x^2 e^x dx$$

Application of integration by parts which leads to an equation.

$$7) \int_0^{\pi/2} e^x \sin x dx \quad 8) \int_1^e \frac{\ln x}{x} dx \quad 9) \int_0^\pi \sin^2 x dx$$

Method of substitution.

$$10) \int_0^{\pi/4} \operatorname{tg} x dx \quad 11) \int_0^4 x \sqrt{x^2 + 9} dx \quad 12) \int_1^e \frac{\cos(\ln x)}{x} dx$$

From examinations.

Various integrals.

$$13) \int_0^{1/2} \arcsin x dx \quad 14) \int_0^{\pi/2} \sin x \cos x dx \quad 15) \int_0^4 e^{\sqrt{x}} dx$$

Area of a plane region. In the following exercises compute areas of plane regions bounded by given curves (graphs of functions).

$$16) y = x^2 - 2x, \quad y = x + 4 \quad 17) y = 2, \quad y = \frac{x^2}{2}, \quad y = x$$
$$18) x = 1, \quad y = e^x, \quad y = e^{-x} \quad 19) y = 0, \quad y = \sqrt{2x+8}, \quad y = \sqrt{2-x}$$

Results.

$$1) \frac{12}{17}(2^{\frac{17}{12}} - 1) \quad 2) 4\sqrt[3]{3} \quad 3) e$$
$$4) \pi \quad 5) 3 \ln 3 - 2 \quad 6) e - 2$$
$$7) \frac{1}{2}(e^{\pi/2} + 1) \quad 8) \frac{1}{2} \quad 9) \frac{\pi}{2}$$
$$10) \frac{1}{2} \ln 2 \quad 11) \frac{98}{3} \quad 12) \sin 1$$
$$13) \frac{\pi}{12} + \frac{\sqrt{3}}{2} - 1 \quad 14) \frac{1}{2} \quad 15) 2(e^2 + 1)$$
$$16) \frac{125}{6} \quad 17) \frac{14}{3} \quad 18) e + \frac{1}{e} - 2$$
$$19) 8$$