9) ARITHMETIC VECTOR SPACE, RANK OF A MATRIX

APPLIED MATHEMATICS (FAPPZ)

Basic. Determine whether the following arithmetic vectors satisfy $\mathbf{u} = \mathbf{v}$.

1) $\mathbf{u} = (5,5,5,5), \mathbf{v} = (5,5,5)$
2) $\mathbf{u} = (\frac{1}{2}, \frac{6}{3}, \frac{3}{2}), \mathbf{v} = (\frac{3}{6}, \frac{18}{9}, \frac{6}{4})$
3) $\mathbf{u} = (\frac{52}{13}, -\frac{85}{17}, 4, \frac{105}{15}), \mathbf{v} = (4, -\frac{65}{13}, \frac{68}{17}, \frac{91}{13})$
4) $\mathbf{u} = (1,2,3), \mathbf{v} = (1,3,2)$

Compute $\mathbf{u} + \mathbf{v}$, $\mathbf{u} - \mathbf{v}$.

5) $\mathbf{u} = (5,-3), \mathbf{v} = (-2,7)$
6) $\mathbf{u} = (\frac{1}{2}, 2, \frac{3}{2}), \mathbf{v} = (1, -\frac{1}{2}, -\frac{3}{2})$
7) $\mathbf{u} = (-7,8,1), \mathbf{v} = (5,-4,7,0)$
8) $\mathbf{u} = (3,-2,7,-1), \mathbf{v} = (1,5,-2,3)$

9) Find the vector $\mathbf{w} = 7\mathbf{u} - \frac{3}{2}\mathbf{v}$ if $\mathbf{u} = (-3,0,5,2)$, $\mathbf{v} = (0,-4,2,50)$.
Find a vector $\mathbf{w}$ which is a linear combination of vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ with coefficients r_1, r_2, r_3.

10) $\mathbf{u}_1 = (1,3), \mathbf{u}_2 = (-1,5), \mathbf{u}_3 = (7,4), \quad r_1 = 1, r_2 = 3, r_3 = 0$
11) $\mathbf{u}_1 = (-7,4,-1), \mathbf{u}_2 = (-1,2,1), \mathbf{u}_3 = (0,-3,1), \quad r_1 = -1, r_2 = 4, r_3 = 5$
12) $\mathbf{u}_1 = (4,1,2,3), \mathbf{u}_2 = (5,3,1,-1), \mathbf{u}_3 = (1,1,1,1), \quad r_1 = 3, r_2 = -2, r_3 = -9$

13) Find a vector $\mathbf{x}$ which satisfies the equation: $2(\mathbf{x} + \mathbf{v}) = 3\mathbf{u} - \frac{1}{2}\mathbf{v}$, kde $\mathbf{u} = (1,-2,2)$, $\mathbf{v} = (2,4,-4)$.

From examinations. 14) Find the rank of the matrix $\begin{pmatrix} 2 & -6 & -7 & 2 \\ 1 & -2 & 4 & -1 \\ 2 & -5 & 1 & -3 \\ -3 & 8 & 3 & -1 \end{pmatrix}$.

15) Find the rank of the matrix $\begin{pmatrix} 1 & 3 & -1 \\ 2 & m & 1 \\ 0 & 5 & m \end{pmatrix}$ in dependance on the parameter m.

Advanced. Decide whether the vector $\mathbf{w}$ is a linear combination of vectors $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$. If it is the case, find this linear combination.

16) $\mathbf{u}_1 = (6,2,2), \mathbf{u}_2 = (7,-3,13), \mathbf{u}_3 = (10,-4,18), \mathbf{w} = (3,1,3)$
17) $\mathbf{u}_1 = (1,2,3), \mathbf{u}_2 = (2,3,3), \mathbf{u}_3 = (1,-2,5), \mathbf{w} = (2,-1,7)$
18) $\mathbf{u}_1 = (1,3,2), \mathbf{u}_2 = (1,1,1), \mathbf{u}_3 = (2,4,3), \mathbf{w} = (4,8,6)$

19) Using the Gaussian elimination determine linear dependence of the vectors $(2,3,-1,2)$, $(3,-1,2,1)$, $(5,2,1,3)$, $(-1,2,-3,2)$.
20) Using the Gaussian elimination determine linear dependence of the vectors $(4,-6,-3)$, $(1,1,-m)$, $(m,-4,-1)$ according to the parameter m.
21) Determine dimension and find a basis of the linear span of the following vectors (i.e. a linearly independent set of its generators)
$\langle (5,5,3,-7), (4,1,3,-5), (-2,0,-1,1), (1,-1,1,-1), (0,3,0,-2) \rangle$.

Results.

1) no

2) yes

3) yes

4) no

5) $\mathbf{u}+\mathbf{v}=(3,4), \mathbf{u}-\mathbf{v}=(7,-10)$

6) $\mathbf{u}+\mathbf{v}=(\frac{3}{2},\frac{3}{2},0), \mathbf{u}-\mathbf{v}=(-\frac{1}{2},\frac{5}{2},3)$

7) operations are not defined

8) $\mathbf{u}+\mathbf{v}=(4,3,5,2), \mathbf{u}-\mathbf{v}=(2,-7,9,-4)$

9) $\mathbf{w}=(-21,6,32,-61)$

10) $\mathbf{w}=(-2,18)$

11) $\mathbf{w}=(3,-11,10)$

12) $\mathbf{w}=(-7,-12,-5,2)$

13) $\mathbf{x}=(-1,-8,8)$

14) $\mathrm{h}=3$

15) $\mathrm{h}=2$ for $m=3\pm 2\sqrt{6}$, otherwise $\mathrm{h}=3$

16) no

17) yes, $\mathbf{w}=\frac{3}{7}\mathbf{u}_1+\frac{8}{7}\mathbf{u}_2$

18) yes, e. g. $\mathbf{w}=2\mathbf{u}_3$

19) linearly dependent

20) independent if and only if $m\neq 2, \frac{1}{6}$

21) e. g. $\{(1,-1,1,-1),(0,1,1,-3),(0,0,3,-7)\}$