INDEFINITE INTEGRAL — BASIC FORMULAE, RULES, METHODS OF INTEGRATION

Table of integrals.

$\int 0 \,\mathrm{d}x = C$		on $(-\infty, \infty)$,
$\int x^\alpha \,\mathrm{d}x = \frac{x^{\alpha+1}}{\alpha+1} + C$	$(\alpha \neq -1)$	on $(0, \infty)$ (or $\mathbb{R}$, or $\mathbb{R} \setminus \{0\}$),
$\int \frac{\mathrm{d}x}{x} = \ln \lvert x \rvert + C$		on $(-\infty, 0)$, or on $(0, \infty)$,
$\int e^x \,\mathrm{d}x = e^x + C$		on $(-\infty, \infty)$,
$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C$	$(a > 0, a \neq 1)$	on $(-\infty, \infty)$,
$\int \sin x \,\mathrm{d}x = -\cos x + C$		on $(-\infty, \infty)$,
$\int \cos x \,\mathrm{d}x = \sin x + C$		on $(-\infty, \infty)$,
$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{cotg} x + C$		on $(k\pi, \pi + k\pi)$, $k \in \mathbb{Z}$,
$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$		on $(-\frac{\pi}{2} + k\pi, \frac{\pi}{2} + k\pi)$, $k \in \mathbb{Z}$,
$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$		on $(-\infty, \infty)$,
$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$		on $(-1, 1)$.

Rules for integration. Let functions f, g have antiderivatives on interval $\mathcal{J}$. Then the functions $(f+g)$, $(f-g)$ and $c\,f$, $c \in \mathbb{R}$, have antidrivatives on $\mathcal{J}$ and

$$\int (f(x) + g(x)) \,\mathrm{d}x = \int f(x) \,\mathrm{d}x + \int g(x) \,\mathrm{d}x,$$
$$\int (f(x) - g(x)) \,\mathrm{d}x = \int f(x) \,\mathrm{d}x - \int g(x) \,\mathrm{d}x,$$
$$\int c f(x) \,\mathrm{d}x = c \int f(x) \,\mathrm{d}x.$$

Methods of integration.

Method of direct integration. We use only basic formulae, rules for integration and manipulation with expressions.

Method of integration by parts.

$$\int u(x)v'(x) \,\mathrm{d}x = \left| \begin{array}{ll} u = u(x) & v' = v'(x) \\ u' = u'(x) & v = v(x) \end{array} \right| = u(x)v(x) - \int u'(x)v(x) \,\mathrm{d}x$$

Change of variables.

- 1st method

$$\int f\big(g(x)\big)\, g'(x) \,\mathrm{d}x = \left| \begin{array}{c} g(x) = t \\ g'(x) \,\mathrm{d}x = \mathrm{d}t \end{array} \right| = \int f(t) \,\mathrm{d}t$$
$$= F(t) + C = F\big(g(x)\big) + C$$

- 2nd method

$$\int f(t)\ \mathrm{d}t = \left| \begin{array}{c} t = g(x) \\ \mathrm{d}t = g'(x)\ \mathrm{d}x \end{array} \right| = \int f\big(g(x)\big)\, g'(x)\ \mathrm{d}x$$
$$= G(x) + C = \left| \begin{array}{c} g(x) = t \\ x = g^{-1}(t) \end{array} \right| = G\big(g^{-1}(t)\big) + C$$

Useful formulae.

$$\int \frac{f'(x)}{f(x)}\ \mathrm{d}x = \ln\big|f(x)\big| + C$$
$$\int \frac{f'(x)}{\big(f(x)\big)^k}\ \mathrm{d}x = \frac{-1}{(k-1)\big(f(x)\big)^{k-1}} + C \quad (k \neq 1)$$